MODELING AND MEASURING THE POLARIZATION OF LIGHT:

FROM JONES MATRICES TO ELLIPSOMETRY

OVERALL GOALS

The Polarization of Light lab strongly emphasizes connecting mathematical formalism with measurable results. It is not your job to understand every aspect of the theory, but rather to understand it well enough to make predictions in a variety of experimental situations. The model developed in this lab will have parameters that are easily experimentally adjustable. Additionally, you will refine your predictive models by accounting for systematic error sources that occur in the apparatus. The overarching goals for the lab are to:

- Model the vector nature of light. (Week 1)
- Model optical components that manipulate polarization (e.g., polarizing filters and quarter-wave plates). (Week 1)
- Measure a general polarization state of light. (Week 1)
- Model and measure the reflection and transmission of light at a dielectric interface. (Week 2)
- Perform an ellipsometry measurement on a Lucite surface.

WEEK 1

PRELIMINARY OBSERVATIONS

Question 1	**Set up the optical arrangement shown in Figure 1.** It consists of (1) a laser, (2) Polarizing filters, (3) A quarter-wave plate (4) A photodetector. Observe the variation in the photodetector voltage as you rotate the polarizing filters and/or quarter-wave plate.

This is as complicated as the apparatus gets. The challenge of week 1 is to build accurate models of these components and model their combined effect in an optical system. An understanding of polarized light and polarizing optical elements forms the foundation of many optical applications including AMO experiments such as magneto-optical traps, LCD displays, and 3D projectors.

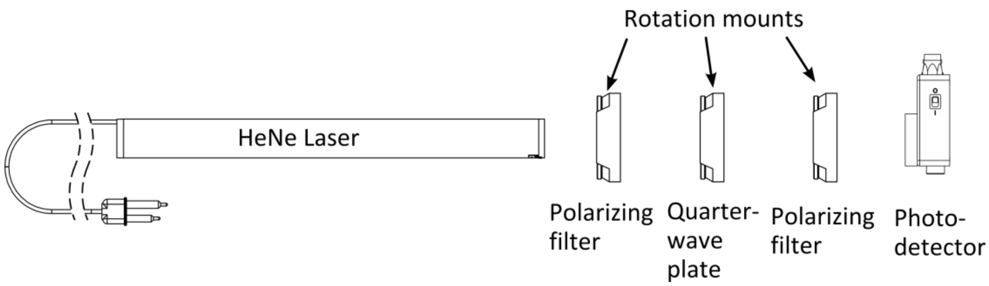

Figure 1: Diagram of a scheme for the measurement of elliptical polarization parameters and the creation of circularly polarized light.

INTRODUCTION

Light is a propagating oscillation of the electromagnetic field. The general principles that govern electromagnetic waves are Maxwell's equations. From these general relations, a vector wave equation can be derived.

$$\nabla^2 \vec{\boldsymbol{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} \tag{1}$$

One of the simplest solutions is that of a plane wave propagating in the $\hat{\boldsymbol{z}}$ direction is

$$\vec{\boldsymbol{E}}(x, y, z, t) = E_x \hat{\boldsymbol{x}} \cos(\omega t - kz + \phi_x) + E_y \hat{\boldsymbol{y}} \cos(\omega t - kz + \phi_y) \tag{2}$$

Where E_x and E_y are the electric field magnitudes of the x-polarization and y-polarization, $\omega = 2\pi f$ is the angular frequency of the oscillating light wave, $k = 2\pi/\lambda$ is the wave-number, and ϕ_x and ϕ_y are phase shifts.

Question 2	Use complex exponential notation to express the plane wave shown in Eq. 2. The real part of this complex expression should match Eq. 2.

Most of our other optics labs assume that light is a *scalar* field, and obeys a scalar wave equation, $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$, but the whole point of this lab is to model and measure the *vector* nature of light.

Question 3	**Reflect**. What experiments and other optics phenomena have you studied in the lab? Did these use a scalar model of light or a vector model?

EXPERIMENT: DETERMINING POLARIZATION OF YOUR LASER

One of the most basic polarization optics is the polarizing filter. An ideal polarizing filter absorbs 100% of one polarization and transmits 100% of an orthogonal polarization. For now we will assume the polarizing filters you have in lab are ideal. Later in the lab we will experimentally develop a model for non-ideal filters which are closer to what we have in lab.

Question 4	Given three things: (1) a laser, (2) a polarizing filter, and (3) and a photodiode: a. Design and carry out a quick experiment to determine if the light emitted by your laser has a well-defined polarization. b. Design and carry out a quick experiment to determine if your photodiode responds equally well to all polarizations of light.

MODELING THE POLARIZATION OF LIGHT WITH JONES VECTORS

If we look back at Eq. (2) we see that only free parameters describing the electric field of a plane wave are the two electric field amplitudes E_x and E_y, and the phases ϕ_x and ϕ_y. In fact, based on your answer to Question 2, it is possible to rewrite the complex exponential form for the electric field as

$$\vec{\boldsymbol{E}}(x,y,z,t) = E_0 e^{i(\omega t - kz)}\left[\cos\theta\ e^{i\phi_x}\hat{\boldsymbol{x}} + \sin\theta\ e^{i\phi_y}\hat{\boldsymbol{y}}\right] \tag{3}$$

Question 5	Show that Eq. (3) follows from your answer to Question 2. Find E_0, and θ in terms of E_x and E_y.

The only thing that is different between different states of polarized light are the complex valued coefficients in front of $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$. In the experiments we are doing this week, we are not concerned with the direction the light is propagating, or the spatial shape of the beam, or the wavelength. If we strip away all the extraneous details of Eqs. (2) and (3), we can write the polarization state of light as a 2x1 vector.

$$E_0 e^{i(\omega t - kz)}\left[\cos\theta\ e^{i\phi_x}\hat{\boldsymbol{x}} + \sin\theta\ e^{i\phi_y}\hat{\boldsymbol{y}}\right] \rightarrow \begin{pmatrix}\cos\theta\, e^{i\phi_x}\\ \sin\theta\ e^{i\phi_y}\end{pmatrix} \tag{4}$$

So, for example, for light polarized purely in $\hat{\boldsymbol{x}}$ or $\hat{\boldsymbol{y}}$ we get

$$\hat{\boldsymbol{x}} \longrightarrow \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \hat{\boldsymbol{y}} \longrightarrow \begin{pmatrix}0\\1\end{pmatrix} \tag{5}$$

These are the two basis polarization states in the Jones matrix notation. Throughout this lab you will be developing the mathematical and computational representations for a model of polarized light based on the Jones Formalism.[1]

Question 6	Suppose two beams of light of different polarization $\begin{pmatrix}E_x\\E_y\end{pmatrix}$ and $\begin{pmatrix}E'_x\\E'_y\end{pmatrix}$ are being combined using a beam splitter. The Jones matrix formalism suggests that the final polarization state after a 50/50 beamsplitter would be proportional to $\begin{pmatrix}E_x + E'_x\\E_y + E'_y\end{pmatrix}$. Under what experimental conditions would this use of the Jones formalism be valid, meaning it would accurately describe the final polarization state of the light?
Question 7	Write a Jones vector in the form of Eq. (4) for linearly polarized light with a polarization angle 45 degrees between $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$.

CONSTRUCTING AND REFINING A MODEL OF A POLARIZING FILTER USING THE JONES FORMALISM

The next few questions will lead us through describing optical components that take a polarization state and turn it into a different polarization state. All of these components can be described by 2x2 matrices.

An ideal polarizer oriented along the x-axis keeps the $\hat{\boldsymbol{x}}$-component unchanged, while the $\hat{\boldsymbol{y}}$-component vanishes because it is not transmitted. Or in the formalism of Jones

$$\begin{pmatrix} \cos\theta\, e^{i\phi_x} \\ \sin\theta\;\; e^{i\phi_y} \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta\, e^{i\phi_x} \\ 0 \end{pmatrix} \tag{6}$$

Question 8	Do a little research in order to explain the basic physics ideas for why the polarizing filter only absorbs one polarization.
Question 9	a. Find the coefficients a, b, c, and d of a 2x2 matrix P_x which describes the behavior of an ideal polarizing filter which transmits only the $\hat{\boldsymbol{x}}$-polarization as given in Eq. (6), so that $\begin{pmatrix} \cos\theta\, e^{i\phi_x} \\ 0 \end{pmatrix} = P_x \begin{pmatrix} \cos\theta\, e^{i\phi_x} \\ \sin\theta\;\; e^{i\phi_y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \cos\theta\, e^{i\phi_x} \\ \sin\theta\;\; e^{i\phi_y} \end{pmatrix}$. b. What is the physical meaning of the diagonal elements a and d? What is the physical meaning of the off-diagonal elements c and b? c. Our actual polarizer is probably not ideal. It doesn't transmit 100% of any polarization, and probably lets a little bit of the orthogonal polarization through. But it probably doesn't transform much light from one polarization into another. So which coefficients in the matrix should we keep to model our polarizer?
Question 10	**Experiment: Refining the idealized model of a polarizing filter to be more realistic.** The ideal polarizing filter transmits 100% of one polarization and 0% of the orthogonal polarization. Is this a good model of the real polarizing sheets we are using in the lab? a. Design and carry out an experiment to measure the maximum and minimum transmission coefficients and construct a more realistic model of the polarizer. b. Write a matrix for a more realistic model of the non-ideal polarizing filter measured in (a). c. Do the polarizing filter characteristics depend on where on the sheet the laser strikes the polarizer? Hint: The power in a optical beam P_{opt} is proportional to the square of the electric field magnitude, $P_{opt} \propto \left|\vec{E}\right|^2 = \left|\begin{pmatrix} E_x \\ E_y \end{pmatrix}\right|^2 = \left|E_x\right|^2 + \left|E_y\right|^2$.

MALUS'S LAW – AN EXPERIMENTAL TEST OF OUR MODEL OF LIGHT AND POLARIZING FILTERS.

Malus's law says the fraction of linearly polarized light transmitted through an ideal polarizer is $I_{trans} = I_0 \cos^2\theta$, where θ is the angle between the incident polarization and the transmitting axis of the polarizer.

Question 11	Briefly explain the basic physics of Malus's law. It is fine to consult a textbook.

If our model of the polarization of light and model of the polarizing filter are a good description, then we should be able to use this model to derive Malus's law in the case of non-ideal polarizing filters. This section will lead you through this modeling exercise.

Briefly, a rotation matrix by an angle theta can be written as

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (7)$$

This matrix can be used to rotate a polarization state, or to rotate an optical element. It will probably be helpful to write express these matrices as functions in Mathematica, which would look like

```
r[th_]:={{Cos[th], Sin[th]}, {-Sin[th],Cos[th]}}
```

A polarizing filter matrix which transmits $\hat{\boldsymbol{x}}$, P_x that is rotated by an angle θ, the rotated polarizer has a matrix given by

$$P(\theta) = R(\theta)P_x R(-\theta) \quad (8)$$

If you want more details on defining functions in Mathematica , there is a YouTube screencast[2] and a Wolfram tutorial[3] available. Mathematica also has many capabilities for handling vectors and matrices, which are documented in the built-in help or on the Wolfram website.[4] In particular, a vector can be represented as `a = {a1,a2}`, a matrix can be represented by `b = {{b11, b12}, {b21, b22}}`. Also, the "dot" operator (a period) is used for multiplication between matrices and other matrices, like `b.b`, and matrices and vectors, like `b.a`.

Question 12	**Using the Jones formalism to predict Malus' law** will give you confidence in more complicated models, like those that include the quarter-wave plate. a. Express in Mathematica the matrix $P(\theta)$ for an ideal polarizing filter at an angle θ. Does $P(\theta = 90°)$ agree with what you expect? b. Use the Jones formalism computational model to predict the transmission between two successive polarizing filters oriented at angles differing by θ. Does it agree with Malus' Law, i.e. $P_{trans} = P_{inc}\cos^2\theta$?
Question 13	**Experimentally test your model of Malus's law** using non-ideal polarizers. Do you get agreement within measurement uncertainties?

MODELING A QUARTER-WAVE PLATE WITH JONES MATRICES

A quarter-wave plate is an optic that transmits both orthogonal polarizations, but the index of refraction is different for the two polarizations. So although they traverse the same physical length, one polarization travels more slowly than the other, and exits the quarter-wave plate with a slightly different phase. Mathematically, we can write a matrix M_{QWP} describing the ideal QWP as

$$M_{QWP} = \begin{pmatrix} e^{i\pi/2} & 0 \\ 0 & 1 \end{pmatrix} \quad (9)$$

It is almost the same as the identity matrix, but for a quarter-wave plate the $\hat{\boldsymbol{x}}$-polarization exits with an additional $\pi/2$ phase shift relative to the $\hat{\boldsymbol{y}}$-polarization. One common application is for creating circularly polarized light.

Question 14	The quarter-wave plate is made of a crystal, commonly quartz. a. What is the difference between the molecular structure of glass and a quartz crystal? b. Could a glass plate act as a quarter-wave plate? Why or why not?

CREATING, MODELING, AND MEASURING ELLIPTICALLY POLARIZED LIGHT

A general state of polarized light is often called elliptically polarized light because the polarization vector has a magnitude and direction that follows an elliptical pattern over time. Two demonstrations on the Wolfram demonstrations website[56] might help you visualize what is going on. We can write this arbitrary polarization state in the following way:

$$\psi = E_0 \begin{pmatrix} \cos\theta \, e^{i\phi_x} \\ \sin\theta \, e^{i\phi_y} \end{pmatrix} \tag{10}$$

It turns out that only the difference between the two phases, $\phi = \phi_x - \phi_y$, is responsible for the elliptical polarization state, so in this lab we will represent an arbitrary elliptical state as

$$\psi' = E_0 \begin{pmatrix} \cos\theta \, e^{i\phi} \\ \sin\theta \end{pmatrix} \tag{11}$$

Question 15	What value(s) θ and ϕ correspond to linearly polarized light? Circularly polarized light?
Question 16	Represent the quarter-wave plate matrix in Mathematica. Use the representation to predict the outgoing state of light when the input polarization has an angle a. 0 degrees to the $\hat{\boldsymbol{x}}$-axis. b. 30 degrees to the $\hat{\boldsymbol{x}}$-axis. c. 90 degrees to the $\hat{\boldsymbol{x}}$-axis. d. For the cases above, describe the polarization of the outgoing light (linear, circular, elliptical)

WEEK 1 GRAND CHALLENGE

1. Design a scheme to measure the parameters of elliptically polarized light: E, θ, and $\phi = \phi_x - \phi_y$
2. Attempt to create circularly polarized light.
3. Model systematic error sources and determine which ones are limiting your ability to produce circularly polarized light.
4. Modify your experiment to create more pure circularly polarized light.

The following set of questions should lead you through this process. Half of your formal written or oral presentation will be explaining this experiment and your results.

<table>
<tr>
<td>Question 17</td>
<td>A measurement of the parameters of elliptical polarization, θ and $\phi = \phi_x - \phi_y$, using a rotatable polarizing filter.
a. Predict the power transmitted through the polarizing filter as a function of the polarizing filter's orientation, θ_{pol} for an arbitrary polarization (θ, ϕ) state in Eq. 11. Mathematica's <code>Manipulate</code>[7] function may be helpful for seeing how the prediction changes as you vary θ and ϕ.
b. Use the predictive function as a fit function for real data. A test data set is available on the CANVAS Lab The polarization parameters used to generate the test data were $\theta = 2.345$, and $\phi_x - \phi_y = 1.07$. Note that your fit may look good, but return different parameters. This could be for a few reasons:
• Changing θ by π only adds a minus sign to the electric field, which doesn't change intensity measurements.
• Changing θ or ϕ by 2π changes gives the exact same field, so adding multiples of 2π doesn't change the measurement.
• Changing $\theta \to -\theta$ is okay if we also change $\phi \to \phi + \pi$ (this also just changes the electric field by a minus sign)
• Changing $\phi \to -\phi$ also gives the same prediction, which is highly significant because it means this simple measurement cannot distinguish between left- and right-handed circular polarizations.</td>
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<td>Question 18</td>
<td>Production of circularly polarized light using polarizing filters and quarter-wave plates.

Figure 1 shows a setup which uses a quarter-wave plate to manipulate the polarization state of light. For two different angles of incident polarization (45 degrees and another of your choosing):
a. Predict the elliptical polarization parameters after the quarter-wave plate, if the light incident on the plate has polarization parameters (θ_{inc} , ϕ_{inc}).
b. Predict the power transmitted through an analyzing polarizer placed after the quarter wave plate, as a function of its angle θ_{pol}.
c. For what angle of incident polarization θ_{inc} on the quarter wave plate do you get circularly polarized light exiting the plate?
d. Take data for this situation. Use the model from 17a to fit for the parameters θ and ϕ of the light incident on the analyzing polarizer to verify how close to circular your light is.

Note: Make sure you consider how you are calibrating the zero of the angles, such as the incident polarization, the quarter-wave plate, and the analyzing polarizer. Clearly describe this in your notebook.</td>
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Question 19	**This question explores the systematic error effects that could limit your ability to produce circularly polarized light.** In an ideal setup, when you aligned the input polarization at 45 degrees from the quarter-wave plate's axis, you would have created perfectly polarized light, and the power measured by the photodetector should not depend on the analyzing polarizer's angle θ_{an}. But you probably didn't get perfectly polarized light. Among the possible idealizations to consider, three can be relaxed and accounted for using your model of polarized light. These idealizations are: 1. The light incident upon the quarter-wave plate is perfectly linearly polarized ($\phi_x - \phi_y = 0$). 2. The light incident upon the quarter-wave plate is exactly 45 degrees from the axis of the quarter-wave plate. 3. The quarter-wave plate adds exactly a $\pi/2$ phase shift between the fast and slow polarizations. Your computation model can predict the result of your measurement in the previous question when these idealizations are violated. For these three idealizations determine the following: a. Predict how a small violation of the idealization would change the result. b. Can you distinguish between the three systematic error sources? c. Could this systematic error source account for non-ideal result? d. Is the violation of the idealization within tolerances on our ability to measure angles, or the specifications on the quarter-wave plate? e. Which error source, if any, is most likely?
Question 20	Can you use your understanding of the systematic error sources to modify your setup to improve the circularity of the light? How and why would you make changes?

WEEK 2

A BASIC MODEL FOR REFLECTION AT A DIELECTRIC INTERFACE

In week 1 we developed a model for describing light that contained information about the polarization, but had no information about the direction of propagation, wavelength, beam profile, etc. In week 2 we are going to study the reflection and transmission of polarization waves at an interface between two dielectrics.

The general principles needed to model the wave propagation and reflection are

1. Maxwell's wave equation given in Eq. (1): $\nabla^2 \vec{\boldsymbol{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2}$
2. Boundary conditions which need to be satisfied between to media
   a. $\epsilon_i \vec{\boldsymbol{E}}_{i,\perp} = \epsilon_t \vec{\boldsymbol{E}}_{t,\perp}$
   b. $\vec{\boldsymbol{E}}_{i,\parallel} = \vec{\boldsymbol{E}}_{t,\parallel}$ The component of the electric field parallel (tangential) to the surface is continuous.
   c. $\vec{\boldsymbol{B}}_{i,\perp} = \vec{\boldsymbol{B}}_{t,\perp}$ The component of the magnetic field normal to surface is continuous.
   d. $\vec{\boldsymbol{B}}_{i,\parallel}/\mu_i = \vec{\boldsymbol{B}}_{t,\parallel}/\mu_t$

In addition to the general principles we need to specify the specific situation where we will apply the general principles listed above. The simplest and most idealized model makes the following assumptions:

1. The interface between the two dielectrics is an infinite plane.
2. The properties of the two dielectric materials are as follows:
    a. The dielectric permittivity in each material is uniform with values of ϵ_i for the incident wave, and ϵ_t in the medium where the transmitted wave propagates.
    b. The magnetic permeability in the two materials is no different from vacuum, so $\mu_i = \mu_t = \mu_0$.
3. The electromagnetic wave is an infinite plane wave with an wave vector $\vec{\boldsymbol{k}}$ which specifies the wavelength and direction of propagation.

Using the general principles (the wave equation for $\vec{\boldsymbol{E}}$ and boundary conditions) in the specific idealized situation above, we can derive the following reflection and transmission coefficients for the two polarizations field amplitudes. (see Hecht *Optics* Sec. 4.6)

For the polarization normal to the plane of incidence (also called s-polarization)

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{12}$$

$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{13}$$

For the polarization parallel to the plane of incidence (also called p-polarization)

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \tag{14}$$

$$t_\parallel = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} \tag{15}$$

Where the transmitted angle θ_t is given by Snell's law

$$n_t \sin\theta_t = n_i \sin\theta_i \tag{16}$$

Question 21 **Pre-lab** Math-Physics-Data Connection	**Draw a diagram which explains the following quantities:** a. Plane of incidence b. Electric field polarization normal to the plane of incidence c. Electric field polarization parallel to the plane of incidence d. θ_t e. θ_i

The power reflection and transmission coefficients can be derived by considering the power that flows in and out of an area A on the surface of the interface. In vacuum, the intensity of light I is related to the electric field amplitude E_0 by

$$I = \frac{c\epsilon_0}{2} |E_0|^2 \tag{17}$$

In a dielectric where the propagation speed is v_1 and the dielectric constant is ϵ_1, the intensity relates to the electric field by

$$I = \frac{v_1 \epsilon_1}{2} |E_1|^2 \tag{18}$$

The reflected power coefficient R is the ratio of reflected and incident powers incident upon an area A on the surface of the interface. It can be related to the amplitude reflection coefficient by

$$R = \frac{P_r}{P_i} = \frac{I_r A \cos\theta_r}{I_i A \cos\theta_i} = \frac{|E_r|^2}{|E_i|^2} = \left|\frac{E_r}{E_i}\right|^2 = |r|^2 \tag{19}$$

where we made use of the fact that speed of propagation v, the dielectric constant ϵ, and the angles of the incident and reflected beams, θ_i and θ_r, are equal.

Similarly, the transmitted power coefficient T is the ratio of transmitted and incident powers incident upon an area A on the surface of the interface. It can be related to the amplitude transmission coefficient by

$$T = \frac{P_t}{P_i} = \frac{I_t A \cos\theta_t}{I_i A \cos\theta_i} = \frac{v_t \epsilon_t |E_t|^2 \cos\theta_t}{v_i \epsilon_i |E_i|^2 \cos\theta_i} = \frac{n_t}{n_i}\left|\frac{E_t}{E_i}\right|^2 \frac{\cos\theta_t}{\cos\theta_i} = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}|t|^2 \tag{20}$$

where we made use of the relation $\epsilon_j = 1/\mu_j {v_j}^2$, the relation $\mu_i = \mu_t = \mu_0$ for typical dielectrics, and $n_t/n_i = v_i/v_t$ relates the velocity of propagation ratio to the index of refraction ratio.

Question 22 Math-Physics-Data Connection	**Prediction of reflection and transmission coefficients (a.k.a "Fresnel equations").** a. Create a computational representation of this model of reflection and transmission at a dielectric interface. In particular, code up functions for the four amplitude coefficients: $r_\perp$, $r_\parallel$, $t_\perp$, and $t_\parallel$, and the four power coefficients: $R_\perp$, $R_\parallel$, $T_\perp$, and $T_\parallel$. b. Use the functions in part (a) to make plots of four *amplitude coefficients* as a function of the incident angle θ_i. c. Use the functions in part (a) to make plots of the four *power coefficients* as a function of the incident angle θ_i. d. For unpolarized incident light, is there a difference in the power of the parallel and normal parts of the reflected light? Can you imagine why polarized sunglasses might be useful if you are spending time on boat on a sunny day? Would you prefer the glasses to block the light that is parallel to the surface of the water (i.e. perpendicular to the plane of incidence) or vice versa?

Question 23	**Take reflection and transmission coefficient data** a. Design a procedure and apparatus to measure the transmission and reflection coefficients, $R_\perp$, $R_\parallel$, $T_\perp$, and $T_\parallel$, as a function of the incident angle. Make sure your procedure includes both cases of measuring both air to Lucite, and Lucite to air reflection/transmission. Figure 2 may help you get started. b. Draw a diagram showing the alignment of the Lucite, rotation state, and incident, reflected, and transmitted laser beams. c. In order to measure the four coefficients, you need to create incident light that is polarized parallel or perpendicular to the plane of incidence. At the Brewster angle, when $\theta_i = \arctan n_t/n_i$, $r_\parallel = 0$, and the reflected light is pure s-polarized (perpendicular to plane of incidence). • Use this information to calibrate the alignment of the initial polarizer. d. Carry your procedure and quantitatively compare your results with the predictions you made in the pre-lab question. • Which data sets had the best agreement? Which had the worst?

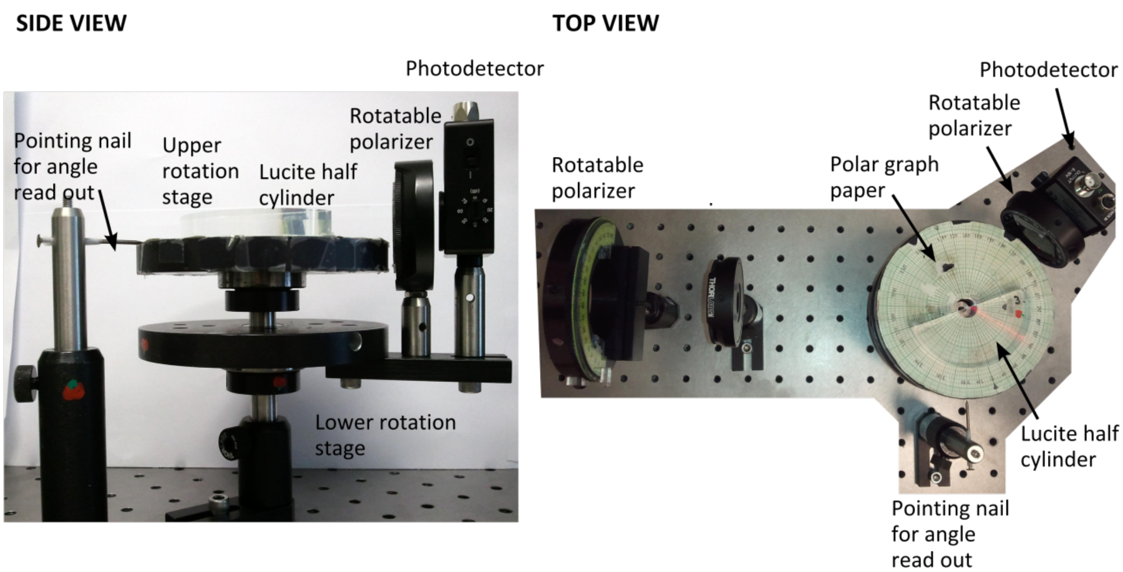

Figure 2: One option for the apparatus used in testing the Fresnel equations. Please ignore the quarter-wave plate shown in the picture on the right after the first rotatable polarizer.

LIMITATIONS AND REFINEMENT OF THE FRESNEL REFLECTION MODEL

In the model of reflection of light from a dielectric interface developed in this lab, many idealizations are made. These include:

1. The incident wave is a plane wave.
2. The medium is uniform.
3. The interface is an infinite plane.
4. The reflection and transmission coefficients only describe the electric fields very close to the interface.

The first assumption, that the incident wave is a plane wave, is only approximately true. We know the laser is well approximated by a Gaussian beam which is composed of a spread of wave-vectors $\vec{\boldsymbol{k}}$. Although there are measurable differences between a plane wave and a Gaussian beam and a plane wave, we are not going to explore this further because it doesn't involve polarization.

The fourth assumption is that the measurement of transmitted and reflected power are only made right at the dielectric interface, but clearly our photodiode is positioned away from the interface.

Hopefully, the next two questions demonstrate that it is important to be explicit about the assumptions in our predictive models, because it gives us hints as to where our experimental results might deviate from our predictions, and how we can refine the models to improve the agreements between prediction and experiement.

Question 24	**Absorption and scattering in the Lucite** a. What effect would absorption and or scattering have in your measurements of the transmission and reflection coefficients? Which of the eight predictions would change? b. What effect would the reflection at the curved dielectric interface have on your measurements? Which of the eight predictions would change? c. How could these deviations be included in the predictions as a fit parameter? Should absorption, scattering, or the second reflection cause an effect which depends on angle? d. Re-plot your data and prediction using the revised model. Do you get better agreement?
Question 25	**Birefringence in Lucite:** **Quick test of the transmitted beam:** Stress in plastics like Lucite cause birefringence, which is the same property of quartz that allows us to engineer quarter-wave plates. However, the birefringence in Lucite is not easy to control, and will vary from sample to sample, and may cause large changes in the transmitted polarization. a. Using simple physics reasoning, for a uniform dielectric medium, how should the incident and transmitted polarization states relate to each other at normal incidence, $\theta_{inc} = 0$? b. Experimentally compare the polarization state of the incident and transmitted beam. How much did it change? c. In Question 19 you measured 8 sets of data for the Air-to-Lucite or Lucite-to-Air reflection and transmission coefficients ($R_{\parallel}$, $R_{\perp}$, $T_{\parallel}$, and $T_{\perp}$). Knowing that birefringence in the Lucite can change the polarization state of the transmitted beam, which of those eight measurements may not be reliable? Why?

GRAND CHALLENGE: ELLIPSOMETRY OF LUCITE

Ellipsometry is a technique used to characterize the thickness and index of refraction of thin films. It is widely used in industrial and academic research. The technique also involves almost everything you have learned from the lab so far, in particular your ability to measure the parameters of an arbitrary polarization state, and the experimental setup for measuring reflection coefficients vs angle.

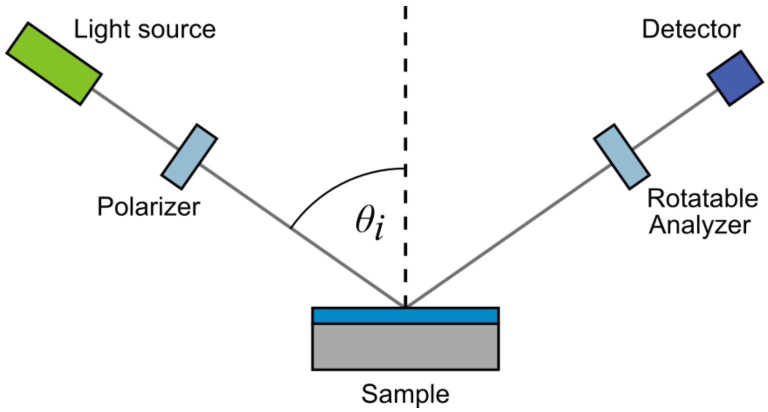

Figure 3: Basic ellipsometry Setup. Modified from http://en.wikipedia.org/wiki/File:Ellipsometry_setup.svg

A standard ellipsometry setup is shown in Figure 3. The standard measurement in ellipsometry is to measure the magnitude and phase of the ratio of the parallel and perpendicular polarization reflection coefficients, i.e.,

$$\rho = \frac{r_\parallel}{r_\perp} = \tan(\Psi)e^{i\Delta} \tag{21}$$

Question 26	**Reflect:** You already took measured reflection coefficients vs. angle of incidence for the air-Lucite interface. Is this data sufficient to determine Ψ and Δ? Why or why not?
Question 27	For the air-Lucite reflection studied in the previous section, use your Fresnel model to predict Ψ and Δ as a function of the angle of incidence θ_i.

The full ellipsometry measurement uses the Jones formalism derived earlier, in particular the measurement of an arbitrary elliptical polarization state. However, instead of choosing $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ as two orthogonal directions for polarization we can use the orthogonal and parallel polarizations to the plane of incidence. The unit vectors for these two polarizations can be represented as vectors:

$$\hat{\boldsymbol{e}}_\perp \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{22}$$

$$\hat{\boldsymbol{e}}_\parallel \to \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23}$$

Question 28	a. If the incident light is polarized 45 degrees from the parallel polarization, what is the Jones vector for the reflected light of the light in terms of $r_\parallel$ and $r_\perp$? b. Using your method from week 1 of determining arbitrary polarizations, you can measure the parameters $E_{refl,45}$, θ, and ϕ for the arbitrary elliptical state of the

	reflected light that was polarized at 45 degrees from parallel polarization $E_{refl,45} = \begin{pmatrix} \cos\theta\, e^{i\phi} \\ \sin\theta \end{pmatrix}$. Find a mathematical relationship between the elliptical polarization coefficients θ and ϕ and the standard ellipsometry coefficients Ψ and Δ defined in Equation 21.
Question 29	Measure Ψ and Δ as a function of the angle of incidence and compare with your predictions. The standard procedure for ellipsometry data is to determine the index of refraction (and thickness, typically) of the sample by finding the set of model parameters that best matches the data. Carry out this analysis for your Lucite data.

If you complete the ellipsometry of Lucite, then congratulations are in order! You have accomplished many scientific tasks, such as developing accurate predictive models, calibrating measurement devices, assembling an apparatus, and carrying out sophisticated data analysis. A natural next step would be measuring thin film layers, which employs similar experimental techniques, but requires more complicated models of reflection from the multilayered surface.

PROJECT IDEAS

1. **Build an ellipsometer**. First, use it to measure the index of refraction of a substrate. Then use it to measure the thickness and index of refraction of a thin film. What is the thinnest film you could measure with this method? Can you measure films thinner than the wavelength of light? What are the performance specifications for measuring Δ and Ψ?
2. **Extending the Jones Calculus to include unpolarized light.** One of the primary limitations of the Jones representation of polarization is that it is only valid for a general state of polarized light, but cannot describe unpolarized, or partially polarized light. This project would extend the model to include partially polarized light, and to create ways to measure the polarization state. It could also explore the connection between describing the polarization of light and the quantum mechanical formalism of a spin ½ system.
3. **Developing a physical model of unpolarized light** This project focuses on modeling unpolarized light as an electromagnetic wave with a randomly fluctuating polarization direction. This project will go in depth into modeling and measuring statistical properties of light. Possible directions include measuring both the coherence time of a single polarization, and the cross-correlation time between two orthogonal polarization states of an unpolarized laser. This can be done without fast electronics, but requires an understanding of interference. Using information learned from these measurements, design a measurement to directly measure polarization fluctuations of the laser. Especially consider the relevant performance specifications of the detectors and electronics.
4. **Polarization behavior of liquid crystals.** Experimentally demonstrate that a liquid crystal can act as a variable retardance wave plate which is controllable by the applied voltage. Use the liquid crystal to phase modulate and amplitude modulate your light, and create a detection scheme for phase and amplitude modulated light. Experimentally test the performance specs of your modulator/demodulator. Encode and decode a fun message or music as a proof of principle demonstration.
5. **What happens to the transmitted beam when the angle is past the critical angle and the reflection goes to 100%?** There is still an electric field where the transmitted beam would be, but it has an exponentially decaying amplitude. Model this exponentially decaying field, called an evanescent wave. Model the coupling of the evanescent wave to some other optical device, like another chunk of glass, or an optical fiber. Measure the coupling of the evanescent field to this optical device as a function of angle of

incidence and separation distance of the device from the interface of reflection. Demonstrate quantitative agreement between your model and measurements.

REFERENCES

[1] Jones, R. C.. "A New Calculus for the Treatment of Optical Systems." *Journal of the Optical Society of America*, 31(7), 488-493. doi:dx.doi.org/10.1364/JOSA.31.000488 (1941)
[2] http://youtu.be/1A4f91yMVhA
[3] http://reference.wolfram.com/mathematica/tutorial/DefiningFunctions.html
[4] http://reference.wolfram.com/mathematica/tutorial/VectorsAndMatrices.html
[5] http://demonstrations.wolfram.com/PolarizationOfAnOpticalWaveThroughPolarizersAndWavePlates/
[6] http://demonstrations.wolfram.com/CircularAndEllipticPolarizationOfLightWaves/
[7] http://reference.wolfram.com/mathematica/ref/Manipulate.html